*J. Fluid Mech.* (1970), *vol.* 44, *part* 3, *pp.* 605–614
*Printed in Great Britain*


# The limiting behaviour of the turbulent transverse velocity component close to a wall

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(Received 4 September 1969 and in revised form 24 March 1970)

Electrochemical techniques are used to measure the circumferential component of the velocity gradient $s_z$ at the wall of a pipe through which a turbulent fluid is flowing. The ratio of the root-mean-squared value of $s_z$ to the time-averaged velocity gradient is found to be 0·09 or 0·1, depending on whether corrections are made for frequency response. The frequency spectrum is similar to that for the component of the wall velocity gradient in the direction of mean flow. The amplitude distribution function for $s_z$ is very roughly approximated by a Gaussian distribution.

## 1. Introduction

Recent studies of turbulent flow close to a wall (Mitchell & Hanratty 1966; Sirkar 1969) have shown how electrochemical techniques, which are the mass transfer analogue of the hot-wire anemometer, can be used to measure the component of the fluctuating wall velocity gradient in the direction of mean flow, $s_x$. In this paper we present the results of experiments in which we use these techniques to measure the component transverse to the direction of mean flow, $s_z$. The component $s_x$ is determined by embedding in the wall circular electrodes or rectangular electrodes with the long side perpendicular to the $x$ axis. Rectangular electrodes at an angle to the $x$ axis are sensitive to flow fluctuations in both the longitudinal and the transverse directions. Mitchell & Hanratty (1966) found that the fluctuating signal from a 45° electrode was only slightly different from the signal of a 90° electrode. The authors (Sirkar & Hanratty 1969) have recently carried out a detailed theoretical study of the effect of the angle of the electrode on its sensitivity to $s_x$ and $s_z$ and found that electrodes at angles of $12\frac{1}{2}°$, 15° and 20° to the $x$ axis are quite suitable to measure $s_z$. From a comparison between the root-mean-squared fluctuating signals from slant electrodes and from 90° electrodes we estimated that $(\overline{s_z^2})^{\frac{1}{2}} \simeq 0{\cdot}087\bar{S}$, where $\bar{S}$ is the time-averaged velocity gradient at the wall. The results of that study encouraged us to attempt a direct measurement of $s_z$ by subtracting the signals from two identical rectangular electrodes arranged in a \ /-configuration with the bisector pointing in the direction of mean flow.

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The results of these measurements of $s_z$ are of particular interest because transverse velocity fluctuations in the immediate vicinity of the wall are much larger than would be suspected from extrapolating hot-wire measurements and play an important role in governing mass transfer at high Schmidt numbers (Sirkar & Hanratty 1970). Measurements with the hot-wire anemometer and with wall electrodes indicate that from the wall to $y^+ \simeq 6$ the fluctuating velocity component in the $x$ direction, $u$, can be represented by the first term in a Taylor series expansion, $u = s_x y$. This is evidenced by finding that a plot of the ratio of the intensity to the local mean velocity $(\overline{u^2})^{\frac{1}{2}}/\overline{U}$ *versus* the distance from the wall extrapolates to a finite value as $y \to 0$ which agrees with the measurements of $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S}$ made with electrochemical techniques. A similar plot of Laufer's (1954) measurements of the transverse velocity fluctuations $(\overline{w^2})^{\frac{1}{2}}/\overline{U}$ *versus* $y$ extrapolates to zero as $y \to 0$. These results suggest either that $s_z = 0$ or that one has to make measurements much closer to the wall to observe the limiting behaviour of $w$ than is necessary for $u$. Our previous study with single slant electrodes (Sirkar & Hanratty 1969) as well as the present study and the measurements of the motions of solid particles close to a wall by Fowles, Smith & Sherwood (1968) indicate that the latter explanation is the better one. The experiments reported in this paper with two slant electrodes provide more reliable measurements of $\overline{s_z^2}$ than have been available and give details about its frequency spectrum and probability distribution.

## 2. Interpretation of electrochemical measurements

Consider the rectangular electrode of length $L$ and width $W$ at an angle $\phi$ to the direction of mean flow shown in figure 1. Assume a uniform flow over the electrode surface. The direction of flow at any instant makes an angle $\theta$ with the direction of mean flow:

$$\tan\theta = -s_z/(\overline{S}+s_x). \tag{1}$$

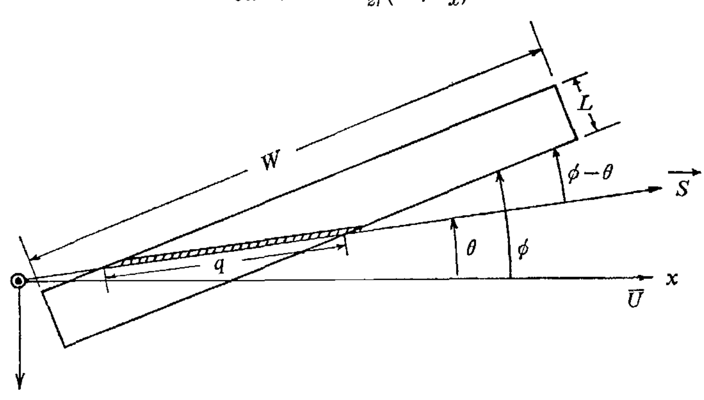


FIGURE 1. Single slant electrode.

Divide the electrode into a number of strips of length $q$ parallel to the instantaneous flow direction. As can be seen from figure 1, the value of $q$ is less near the edges of the electrode than in the central regions. Because of the thinness of the concentration boundary layer, molecular diffusion can be neglected in the

$x$ direction and in the $z$ direction. If the pseudo-steady-state approximation (Mitchell & Hanratty 1966) is valid the mass transfer coefficient for each of these strips equals $\sigma[S/q]^{\frac{1}{3}}$, where $\sigma = 1{\cdot}5D^{\frac{2}{3}}/9^{\frac{1}{3}}\Gamma(\frac{4}{3})$. If $\phi-\theta \geqslant \psi$, where $\psi = \tan^{-1}(L/W)$, the average mass transfer coefficient for the whole electrode at any instant is given as

$$K = \sigma\left[\frac{S\sin(\phi-\theta)}{L}\right]^{\frac{1}{3}}\left[1+\frac{L}{5W}\cot(\phi-\theta)\right]. \tag{2}$$

The term containing $L/W$ appears because $q$ is varying near the edges of the electrode. As $L/W \to 0$ this edge effect can be neglected.

Equation (2) can be expanded under the restriction

$$\frac{|s_z|\cot\phi}{|\bar{S}+s_x|} < 1 \tag{3}$$

to give

$$K = \sigma\left(\frac{\bar{S}\sin\phi}{L}\right)^{\frac{1}{3}}\left[1+\frac{L\cot\phi}{5W}+\frac{1}{3}\frac{s_x}{\bar{S}}+\frac{s_z\cot\phi}{3\bar{S}}+\frac{s_xL\cot\phi}{15\bar{S}W}-\frac{2Ls_z\cot^2\phi}{15W(\bar{S}+s_x)}+\ldots \tag{4}$$

The time-averaged mass transfer coefficient is then given as

$$\bar{K} = \sigma\left(\frac{\bar{S}\sin\phi}{L}\right)^{\frac{1}{3}}\left(1+\frac{L\cot\phi}{5W}\right). \tag{5}$$

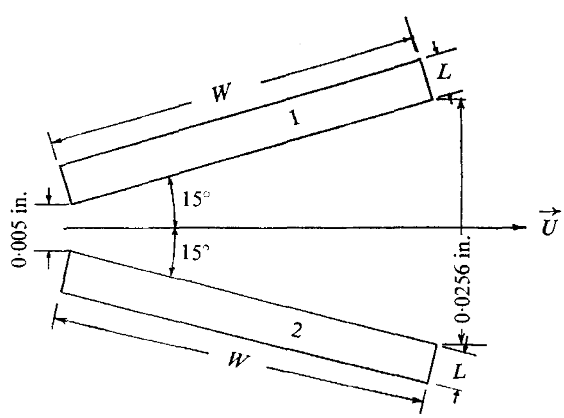


FIGURE 2. Electrode dimensions. $L = 0{\cdot}003$ in., $W = 0{\cdot}040$ in.

The mass transfer coefficients to the two electrodes shown in the \ / arrangement in figure 2 are

$$K_1 = \bar{K}+\sigma\left(\frac{\bar{S}\sin\phi}{L}\right)^{\frac{1}{3}}\left[\frac{1}{3}\frac{s_x}{\bar{S}}\left(1+\frac{L\cot\phi}{5W}\right)+\frac{1}{3}\frac{s_z}{\bar{S}}\left(\cot\phi-\frac{2L\cot^2\phi}{5W}\right)\right], \tag{6}$$

$$K_2 = \bar{K}+\sigma\left(\frac{\bar{S}\sin\phi}{L}\right)^{\frac{1}{3}}\left[\frac{1}{3}\frac{s_x}{\bar{S}}\left(1+\frac{L\cot\phi}{5W}\right)-\frac{1}{3}\frac{s_z}{\bar{S}}\left(\cot\phi-\frac{2L\cot^2\phi}{5W}\right)\right]. \tag{7}$$

Since

$$K_1-K_2 = \frac{2}{3}\frac{s_z}{\bar{S}}\left(\cot\phi-\frac{2L\cot^2\phi}{5W}\right), \tag{8}$$

we can study the $s_z$ fluctuations by measuring the difference of the electric signals from two identical electrodes in the arrangement shown in figure 2.

The restriction (3) implies that, if $\cot\phi = 5$ ($\phi = 11^\circ\, 18'$), (4) is valid if

$$s_z/(\bar{S}+s_x) < 0{\cdot}2.$$

## 3. Description of experiments

The experiments were conducted on a fully developed turbulent flow in a pipe with an inside diameter of 7·615 in.

The flow loop shown in figure 3 covers five levels. The test fluid discharges from a stainless steel storage tank on the top level to a stainless steel centrifugal pump on the ground level. The pump discharges into a stainless steel calming section which consists of (1) a 6 in. standard 90° elbow with turning vanes, (2) a 3 ft. diffuser with a square downstream section, 22 in. × 22 in., (3) an 8 in. long honeycomb with square cells, (4) a 9 in. long settling chamber, (5) a 20-mesh wire screen, (6) another 9 in. long settling chamber, (7) a 22 in. nozzle having an outlet diameter of 7·625 in. The uniform flow that emerges from the nozzle is tripped by a $\frac{1}{2}$ in. long ring consisting of a series of $\frac{3}{8}$ in. equilateral triangles around the circumference of a 7·625 in. polyvinyl chloride pipe. The test section, which consists of a 29 in. long 7·615 in. cast acrylic pipe, is preceded by a 43 ft. run (67·5 pipe diameters) of 7·625 in. polyvinyl piping and is followed by a 29 in. long section of cast acrylic piping. The end of the acrylic test section is gradually tapered in order to provide a smooth joint with the entry section. The test electrode is located near the downstream end of the test section. All of the auxiliary piping in the flow loop is fabricated either from plastic or from stainless steel. Valves are rubber lined or are plastic.

The flow loop is located in an air-conditioned portion of the building where the temperature is kept at 25 °C. The temperature of the test fluid in the storage tank is controlled at $25 \pm 0{\cdot}1$ °C by stainless steel cooling coils located in the storage tank. Before a run, nitrogen was bubbled into the storage tank while circulating fluid in the flow loop. The storage tank was kept under an atmosphere of nitrogen during the run.

The dimensions of the test electrode are given in figure 2. For this arrangement $\phi = 15^\circ$, $L = 0{\cdot}003$ in., and $W/L = 13{\cdot}33$. The maximum lateral spread of these electrodes was 0·025 in. and was smaller than one half the lateral scale of the $s_x$ fluctuations measured by Mitchell & Hanratty (1966).

Mitchell & Hanratty (1966) have analyzed the frequency response of a rectangular electrode oriented at 90° to the direction of mean flow and found that the pseudo-steady-state approximation is valid provided the dimensionless frequency $\tilde{n} = 2\pi n(L^2/D\bar{S}^2)^{\frac{1}{3}}$ is less than 0·40. Here $D$ refers to the diffusion coefficient. The response of a slant electrode can be estimated by taking the instantaneous length of the electrode in the above dimensionless frequency as $L/\sin\phi$. The values of $\tilde{n}$ corresponding to the average frequencies defined by (9) are found to be less than 0·40 for the range of variables covered in this investigation.

The \\ / electrodes were fabricated in a plug which was mounted flush with the pipe wall. A $\frac{1}{2}$ in. diameter acrylic cylinder $\frac{7}{8}$ in. long was used. A length of $\frac{1}{2}$ in. was turned to $\frac{1}{4}$ in. diameter and another length of $\frac{1}{4}$ in. was given a 60° taper.

Two 0·005 in. wide slots of the required depth were cut in the plug at 30° to each other in a milling machine. Two pieces of platinum $\frac{3}{4}$ in. long × $\frac{1}{4}$ in. wide × 0·003 in. thick were glued into the slots with epoxy resin. The electrodes in the $\frac{1}{4}$ in. end of the plug were machined to give the desired value of $W$ and the

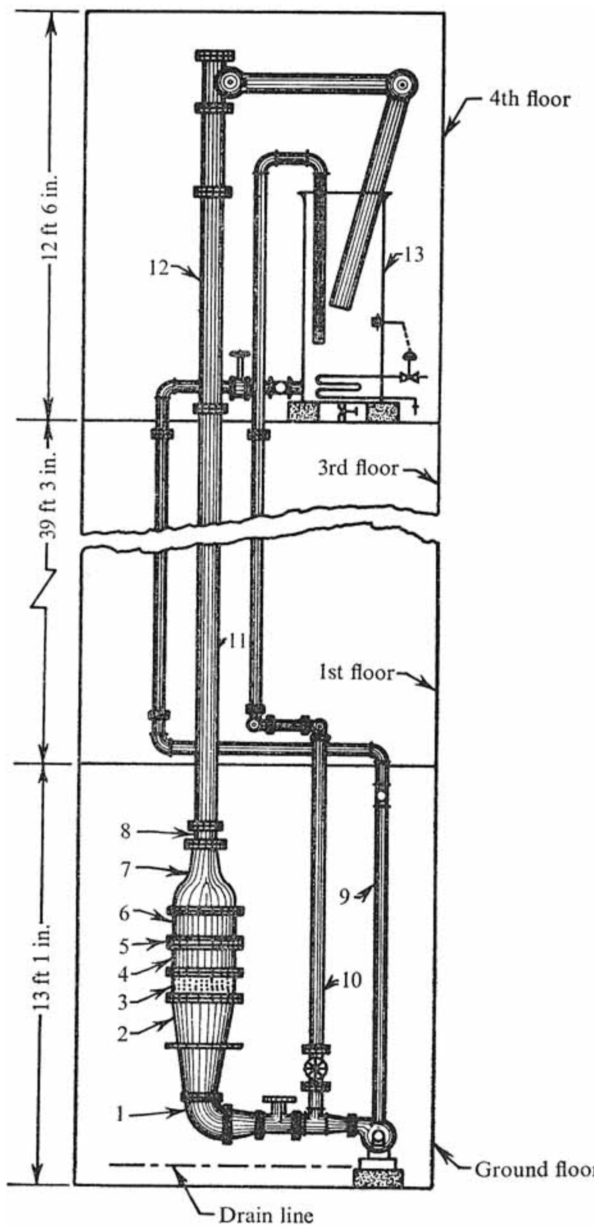


FIGURE 3. 8 in. flow loop elevation. 1, 90° elbow with turning vanes; 2, diffuser; 3, honeycomb; 4, settling chamber $a$; 5, gauge screen; 6, settling chember $b$; 7, contraction nozzle; 8, turbulence trip; 9, downflow, 4 in. sch. 80 UPVC pipe; 10, pump bypass, 4 in. sch. 80 UPVC pipe; 11, upflow entrance length 8 in. pipe; 12, test section; 13, tank. Total length of 8 in. pipe before test section = 43 ft. Test section length = 6 ft. 6 in.

machined end of the plug was filled with epoxy resin. After hardening, the end was machined and polished to produce a set of \\ / electrodes flush with the plug surface. The dimensions of each electrode were determined by a microscope whose eyepiece had an engraved scale calibrated with an American Optical micrometer slide. Several plugs were made and the one for which the dimensions of the two electrodes were nearly identical was selected for experimentation. A $\frac{1}{4}$ in. hole was drilled into the acrylic test section and the plug was cemented into the hole. After the cement had hardened the surface of the plug was sanded smooth with progressively finer grades of emery paper.

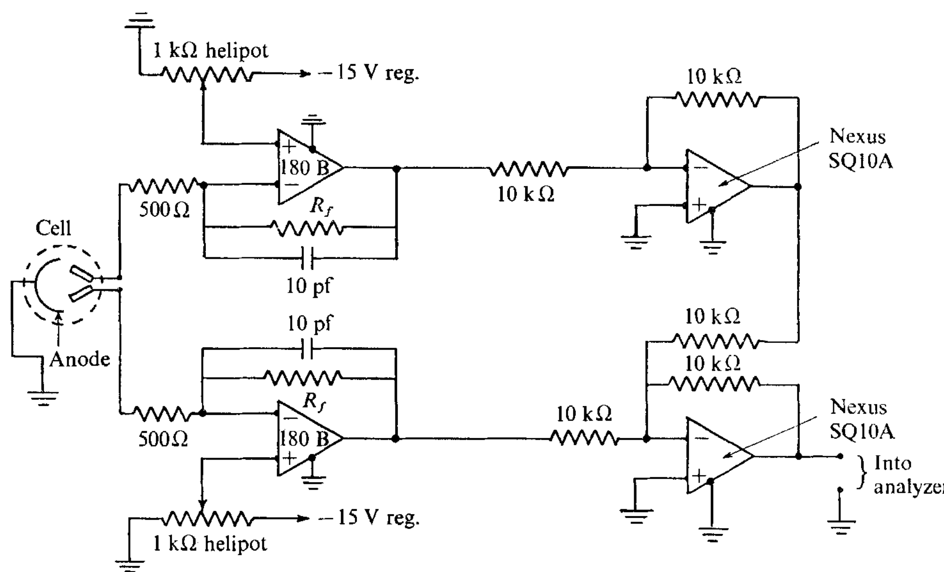


FIGURE 4. Circuitry for \\ / electrodes.

Two electronic circuits similar to those that were used to measure $s_x$ (Mitchell & Hanratty 1966) were used with the two electrodes in the \\/ arrangement (see figure 4). These circuits convert the current fluctuations in the electrochemical circuit to voltage fluctuations at the output of 180 B Analog Devices differential operational amplifiers while keeping the cathode at a constant voltage. The signal from one of these circuits was inverted by a gain of one Nexus SQ 10 A operational amplifier and added to the signal from the other electrode with another Nexus SQ 10 A operational amplifier.

## 4. Results

Measured values of the intensity of the $s_z$ fluctuations that have not been corrected for frequency response are shown in figure 5. We conclude from these measurements that $(\overline{s_z^2})^{\frac{1}{2}}/\overline{S} \simeq 0{\cdot}09$.

In figure 6 relative intensities of the $w$ fluctuations $(\overline{w^2})^{\frac{1}{2}}/\overline{U}$ obtained from the studies of Fowles *et al.* (1968) and of Laufer (1954) are plotted as a function of $y^+$.

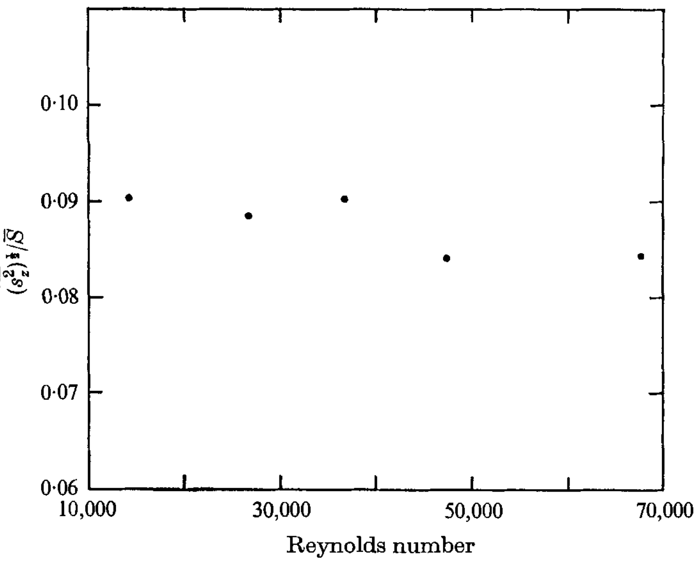


FIGURE 5. Relative intensity of $s_z$ fluctuations from \ / electrodes. ●, data for \ / electrodes.

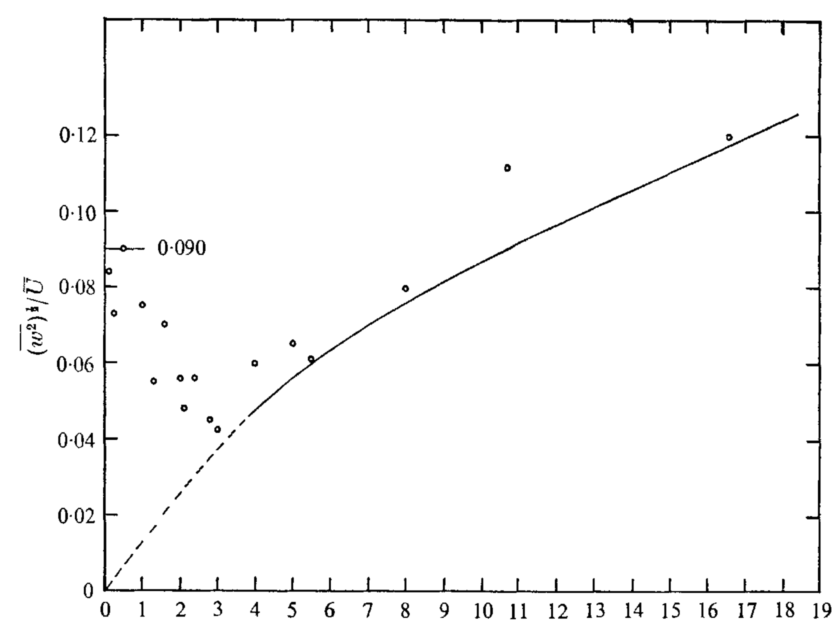


FIGURE 6. Relative intensity of $w$ velocity fluctuations in the wall region. ○, Fowles *et al.* (1968); ——, Laufer (1954); – – –, trend of Laufer's data.

Our measured value of $(\overline{s_z^2})^{\frac{1}{2}}/\bar{S}$ is indicated in the figure as the $\lim_{y^+\to 0} (\overline{w^2})^{\frac{1}{2}}/\bar{U}$. It is clear that the dashed extrapolation of Laufer's measurements would be incorrect. The data of Fowles *et al.* indicate that the plot of $(\overline{w^2})^{\frac{1}{2}}/\bar{U}$ goes through a minimum. This point deserves further investigation since it is possible that errors existed in the measurements of Fowles *et al.* and Laufer below $y^+ = 10$.

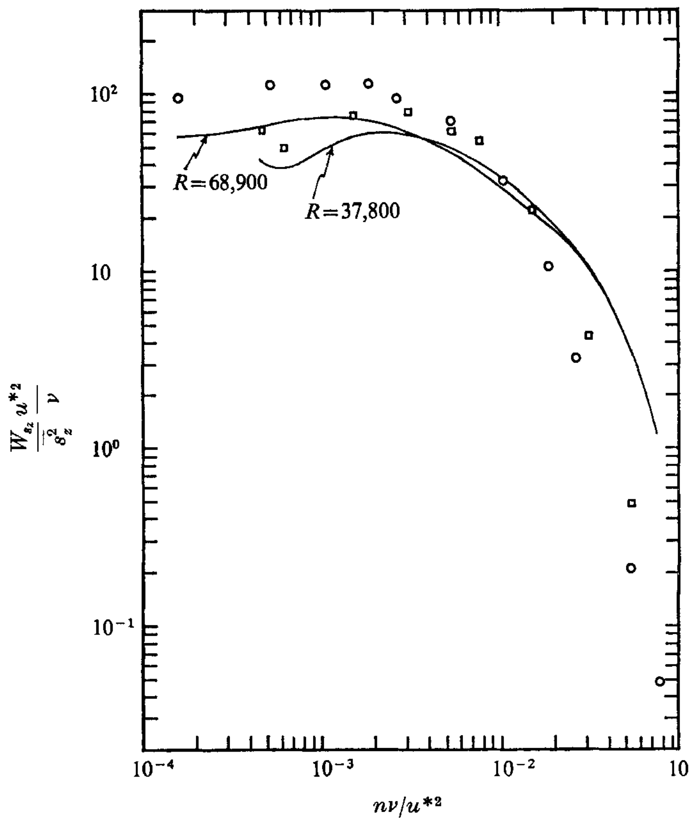


FIGURE 7. Spectra of $s_z$ fluctuations. ○, $R$ = 68,900; □, $R$ = 37,800; ——, corrected spectra.

The measured spectral density functions $W_{s_z}(n)$ at Reynolds numbers of 68,900 and 37,200 are shown as a function of the dimensionless frequency $n\nu/u^{*2}$ in figure 7, where $\nu$ is the kinematic viscosity, $u^*$ is the friction velocity, and $n$ is the frequency in cycles per second. The average dimensionless frequencies $\langle n\nu/u^{*2}\rangle$, where

$$\langle n\rangle = \int_0^\infty nW_{s_z}(n)\,dn \Big/ \int_0^\infty W_{s_z}(n)\,dn, \tag{9}$$

are $7{\cdot}8\times 10^{-3}$ for $R$ = 68,900 and $1{\cdot}1\times 10^{-2}$ for $R$ = 37,800. Since the kinematic viscosity of the fluid used in these studies was $1{\cdot}26\times 10^{-2}\,\text{cm}^2\,\text{sec}^{-1}$, these would correspond to frequencies of 3 and 1·6 c/s. The characteristic frequency of the $s_z$ fluctuations is close to that of the $s_x$ fluctuations. For example $\langle n\nu/u^{*2}\rangle$ for $W_{s_x}(n)$ has been found to be $7{\cdot}6\times 10^{-3}$ at $R$ = 68,900 and $8{\cdot}5\times 10^{-3}$ at $R$ = 37,200

(Sirkar 1969). The high-frequency data points shown in figure 7 are believed to be in error because of the inadequacy of the pseudo-steady-state approximation. Mitchell & Hanratty (1966) have suggested a method for correcting for frequency response whereby

$$\frac{W_{s_z}(n)}{\overline{s_z^2}} = \left(\frac{W_k(n)}{\overline{k^2}}\right) f(\tilde{n}), \tag{10}$$

and $f(\tilde{n})$ is a function which approaches unity at small $\tilde{n}$. Fortuna (1970) has recently evaluated $f(\tilde{n})$ for a wide range of values of $\tilde{n}$. The solid lines in figure 7 are the spectra corrected by using the results of Fortuna. These corrected spectra indicate that the value of $(\overline{s_z^2})^{\frac{1}{2}}/\overline{S} \simeq 0{\cdot}09$ shown in figure 5 could be too low and should be about 0·1. In this calculation the effective length used to evaluate $\tilde{n}$ was taken at $L/\sin\phi$ and the diffusivity as $5{\cdot}25 \times 10^{-6}\,\mathrm{cm^2/sec}$. This correction is not exact since the effective length of the electrode actually varies with time, depending on the instantaneous value of $s_z$.

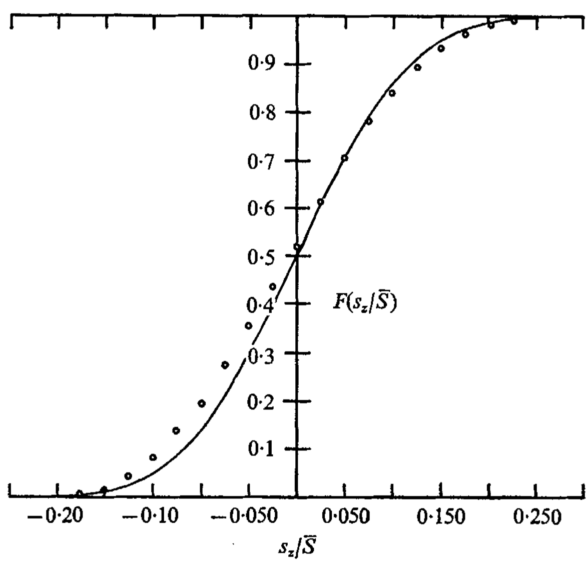


FIGURE 8. Amplitude distribution function of $s_z$ fluctuations. ○, $R = 37{,}800$; ——, Gaussian curve.

The amplitude distribution function $F(s_z/\overline{S})$ defined as

$$F(s_{z1}/\overline{S}) = \frac{\text{time}(s_z/\overline{S} < s_{z1}/\overline{S})}{\text{total time}} \tag{11}$$

was determined from the uncorrected mass transfer measurements and is shown in figure 8. The solid line for a Gaussian distribution with a root-mean-squared value of $s_z/\overline{S}$ of 0·09 is shown for comparison.

This work is supported by the National Science Foundation under Grant NSF GK-2813X.

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